HORNSBY GIRLS HIGH SCHOOL

Mathematics Advanced

Year 12 Practice HSC Examination Term 3 2021

STUDENT NUMBER: ______________________________

General Instructions:

- Reading time – 10 minutes
- Working time – 3 hours
- Write using black pen
- Calculators approved by NESA may be used
- For questions in Section II, show relevant mathematical reasoning and/ or calculations
- A NESA reference sheet is provided

Total Marks: 100

Section I – 10 marks (pages 2–6)

- Attempt Questions 1– 10
- Allow about 15 minutes for this section

Section II – 90 marks (pages 9 - 33)

- Attempt Questions 11 - 31
- Allow about 2 hours and 45 minutes for this section

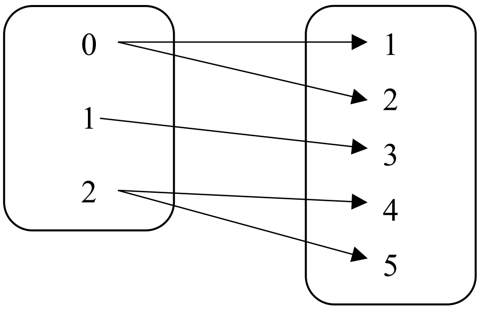
Section I

10 marks
Attempt Questions 1 – 10
Allow about 15 minutes for this section

Use the Objective Response answer sheet for Questions 1 – 10

---

1 What type of relation is shown below?



(A) One-to-One

(B) One-to-Many

(C) Many-to-One

(D) Many-to-Many

2 If $\cos\theta = \frac{-2}{3}$ and $\pi < \theta < \frac{3\pi}{2}$, what is the value of $\tan\theta$?

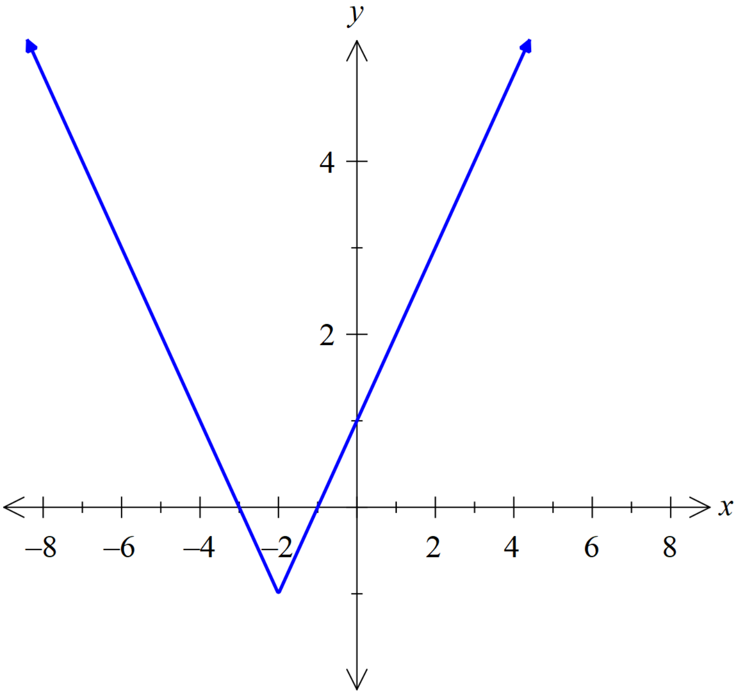
(A) $\frac{\sqrt{5}}{2}$

(B) $\frac{2}{\sqrt{5}}$

(C) $-\frac{\sqrt{5}}{2}$

(D) $-\frac{2}{\sqrt{5}}$

3 The graph of $y = |x+2| - 1$ is shown below.



The solution to the inequality $|x+2| - 1 > 3$ is:

(A) $-1 < x < 1$

(B) $x < -1$ or $x > 1$

(C) $-6 < x < 2$

(D) $x < -6$ or $x > 2$

4 The third term of an arithmetic series is 8 and the sixth term is 23.
The first term of the series is

(A) 0

(B) –2

(C) 2

(D) 3

5 Given that $f(x) = 3x + 1$ and $g(x) = x^3$, evaluate $f(g(2))$

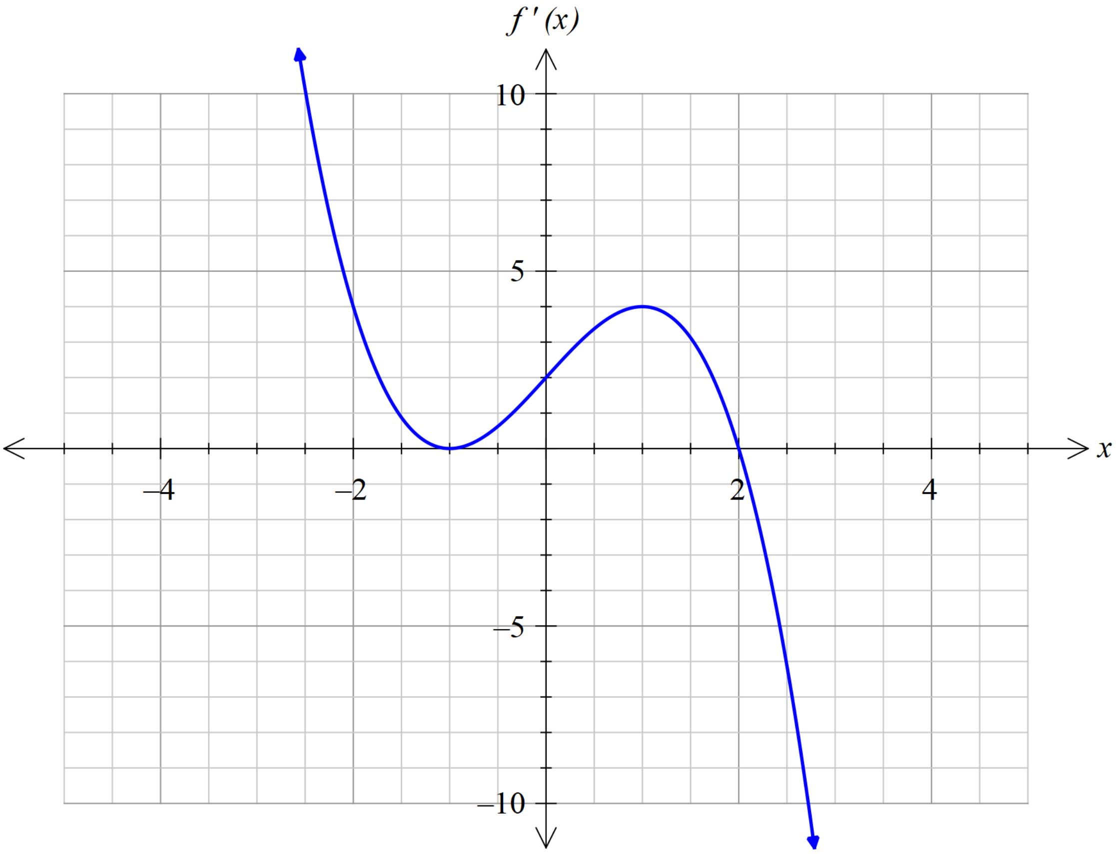
(A) 2

(B) 7

(C) 25

(D) 343

6 The graph of $y = f'(x)$ is shown below.



The values of x for which $f(x)$ is increasing is are:

(A) $x < 2$

(B) $x \leq 2$

(C) $x < -1$ or $-1 < x < 2$

(D) $x = -1$ or $x \geq 2$

7 The relation $\left(x-1\right)^2+y^2=4$ is transformed to $\left(x-3\right)^2+\left(y+1\right)^2=4$ by a horizontal translation followed by a vertical translation.

Which row of the table shows the correct direction and distance of the translation?

	Horizontal Translation	*Vertical Translation*
(A)	Shift right 2 units	Shift down 1 unit
(B)	Shift left 2 units	Shift down 1 unit
(C)	Shift right 2 units	Shift up 1 unit
(D)	Shift left 2 units	Shift up 1 unit

8 If $y=x\left(x^2+1\right)^3$ then $\dfrac{dy}{dx}=$

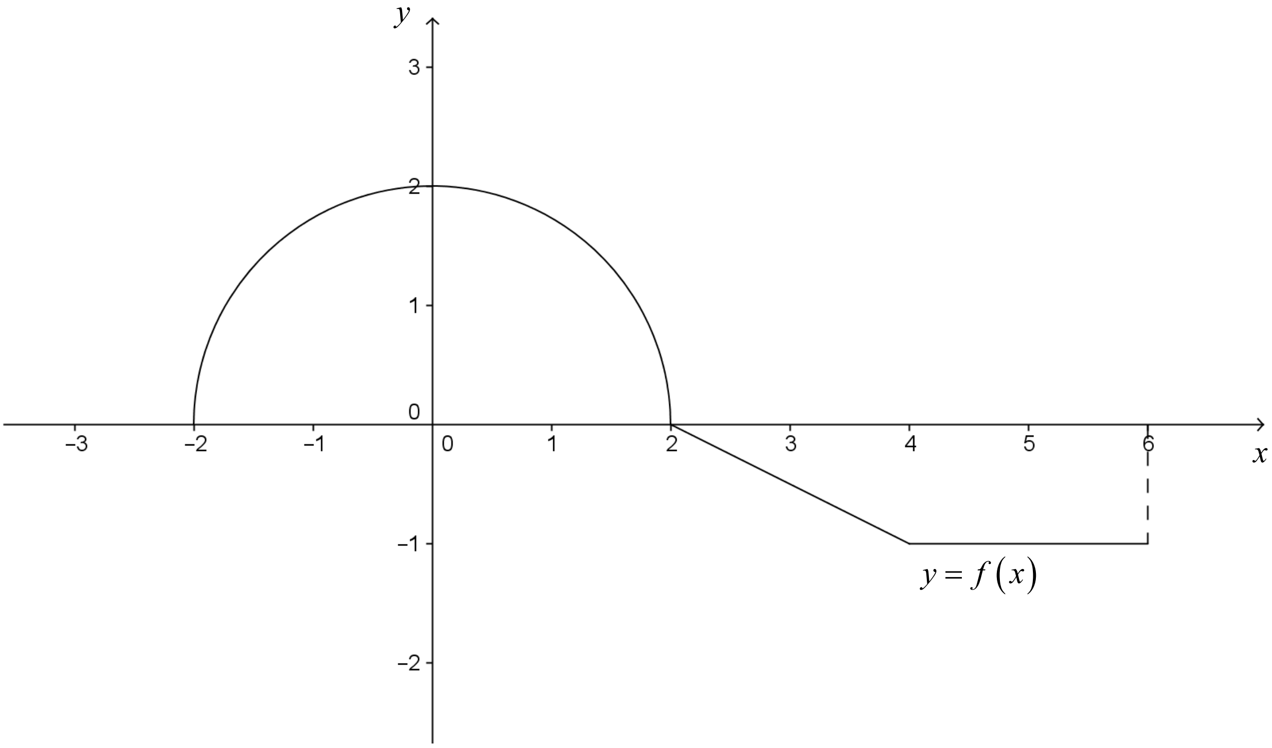
(A) $\left(x^2+1\right)^3+3x\left(x^2+1\right)^2$

(B) $\left(x^2+1\right)^3+3x^2\left(x^2+1\right)^2$

(C) $\left(x^2+1\right)^3+6x^2\left(x^2+1\right)^2$

(D) $y=1+6x\left(x^2+1\right)^2$

9 Using the diagram below, the exact value of $\int_{-2}^{6} f(x)\,dx$ is:



(A) $2\pi - 3$

(B) $2\pi + 3$

(C) $4\pi - 3$

(D) $4\pi + 3$

10 The graph of $y = f(x)$ has a vertical asymptote at $x = 1$ and a horizontal asymptote at $y = 2$.

The graph of $y = -f(-x)$ has a vertical asymptote and horizontal asymptote respectively:

(A) $x = 2$, $y = 1$

(B) $x = -1$, $y = 2$

(C) $x = -1$, $y = -2$

(D) $x = 1$, $y = -2$

End of Section I

Question 11 **(2 marks)**

Rationalise $\dfrac{\sqrt{5}+1}{\sqrt{5}-2}$, giving your answer in simplest form. **2**

Question 12 **(2 marks)**

Find the equation of the tangent to $f\left(x\right)=e^{2x}$ at the point where $x=\dfrac{1}{2}$. **2**

Question 13 **(3 marks)**

Differentiate the following functions

(a) $f\left(x\right)=\tan 2x$ **1**

(b) $f\left(x\right)=\dfrac{e^{x}}{\cos x}$ **2**

Question 14 (4 marks)

(a) Find $\int \frac{x^4 + 3x^2 + 1}{x^2} dx$. **2**

(b) Find $\int \frac{4x}{x^2 + 1} dx$. **2**

Question 15 (3 marks)

A circular disc is cut into twelve sectors whose angles are in an arithmetic sequence. **3**

The angle of the largest sector is twice the angle of the smallest sector.

Find the size of the angle, in degrees, of the smallest sector.

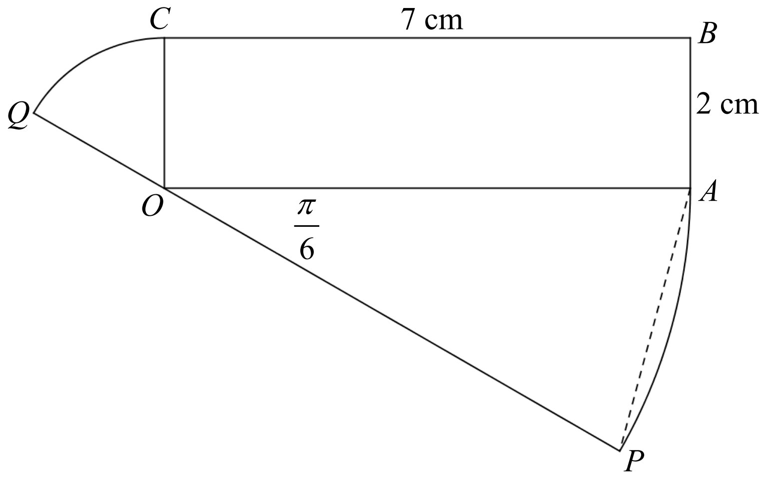
Question 16 (3 marks)

(a) Given that $\log_2 x + \log_2 (x-3) = 2$, show that $x^2 - 3x - 4 = 0$. **2**

(b) Hence, find the solution(s) of the equation $\log_2 x + \log_2 (x-3) = 2$. **1**

Question 17 (6 marks)

In the diagram below, *OABC* is a rectangle with sides 7 cm and 2 cm. *PQ* is a straight line. *AP* and *CQ* are circular arcs, with the centre *O* the centre of both circles, and $\angle AOP = \frac{\pi}{6}$.



NOT TO SCALE

(a) Find the exact arc length *CQ*. **2**

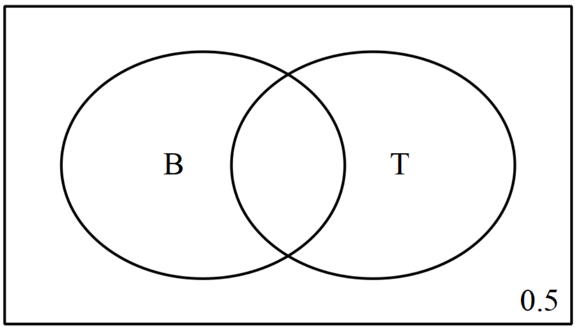
(b) Find the exact area of the sector *OAP* . **2**

(c) Find the exact length of interval *AP*, correct to 1 decimal place. **2**

Question 18 (5 marks)

The probability that a student plays badminton is 0.3. The probability that a student plays neither table tennis or badminton is 0.5, and the probability that a student plays both sports is x.

(a) Using the Venn diagram, or otherwise, complete the probabilities below. **1**



$P(B) =$ _______

$P(B \cap T) =$ ____

(b) Find the probability that a student plays table tennis but not badminton **1**

It is known that if a student plays table tennis, the probability that they also play badminton is 0.5

(c) Find the probability that a student plays both badminton and table tennis. **2**

(d) Hence, or otherwise, determine the probability that a student plays only badminton. **1**

Question 19 (4 marks)

The table below shows the population and income per capita of ten countries from the same economic region.

Country	1	2	3	4	5	6	7	8	9	10
Population, x (millions)	4.49	14.86	7.5	3.76	2.59	9.15	6.70	10.18	4.67	7.95
Income per capita, y (USD)	682	189	353	668	950	266	355	230	491	287

(a) Find the value of r, the Pearson's correlation coefficient, correct to 2 decimal places. **1**

(b) Find the equation of the line of best fit, correct 2 decimal places, using the variables given in the table. **2**

(c) Use your equation to estimate the income per capita of a country from the same economic region with a population of 5.5 million. **1**

Question 20 (5 marks)

A particle moves such that its velocity, v ms^{-1}, at a time t seconds is given by $\dot{x} = 4 - 8\cos t$.
The particle is initially at the origin.

(a) Find the two times the particle is first at rest? **2**

(b) Find the equation for displacement x in terms of time t. **1**

(c) Find the exact distance travelled by the particle between the first two times it is at rest. **2**

Question 21 (5 marks)

The population of an island is increasing according to the formula $P = 20 + 10e^{\frac{t}{50}}$ where P is the population in **thousands** and t is the time in years after January 1st 2020.

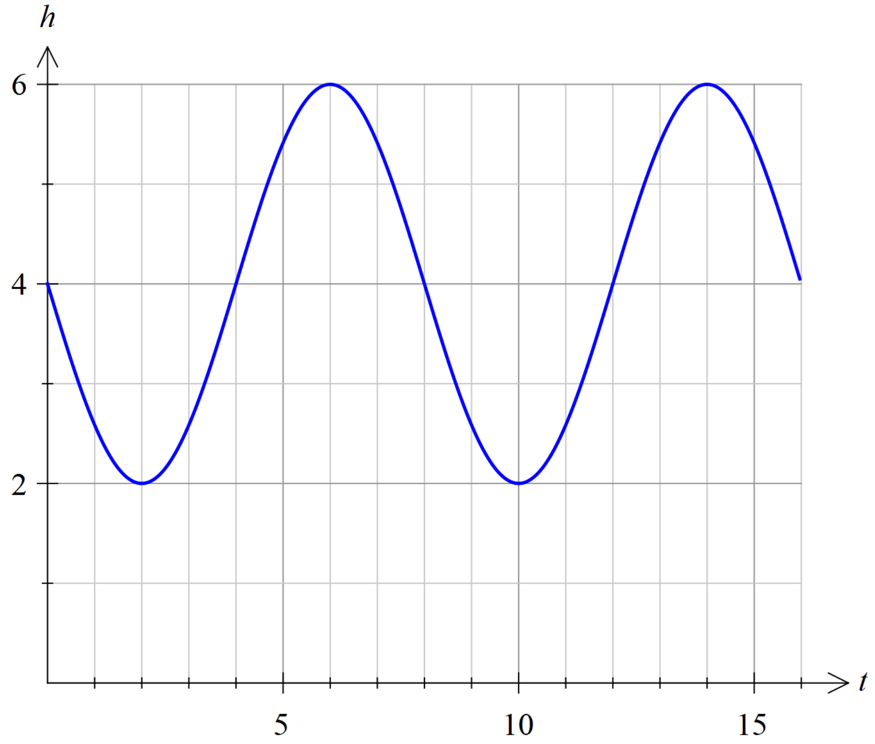
(a) Find the initial rate of change of the population of the island. **2**

(b) Use the model to predict the population on 1st January 2040, correct to the nearest thousand. **1**

(c) Using the model, predict in what year will the population reach 40 000. **2**

Question 22 (4 marks)

The height of water h metres in a harbour can be modelled using the function below:
$h = 4 - A\sin(bt)$, where t is the time in hours after midnight.



At 2 am the height of water in the harbour is at its minimum.
At 6 am the height of water in the harbour is at its maximum.

(a) Find the values of A and b. **2**

A boat can only enter the harbour when the height of the water is more than 5 metres.

(b) Express the information given as an equation, and solve the equation to find the first time the boat can enter the harbour. **2**

Question 23 (2 marks)

Use the trapezoidal rule with 4 sub-intervals to estimate $\int_0^2 \sqrt{x}dx$, correct to 2 decimal places. **2**

Question 24 (3 marks)

(a) Show that the equation $2\cos^2 x = 4 - 5\sin x$ may be written as $2\sin^2 x - 5\sin x + 2 = 0$. **1**

(b) Hence solve $2\cos^2 x = 4 - 5\sin x$, for $0° \leq x \leq 360°$. **2**

Question 25 (4 marks)

The line $y = 24(x-1)$ is tangent to the curve $y = ax^3 + bx^2 + 4$ at $x = 2$.

(a) Using the fact that the tangent meets the curve to show that $2a + b = 5$. **2**

(b) By finding another relationship between *a and b,* find the values of *a* and *b*. **2**

Question 26 (13 marks)

Two children, Jack and Jill, each throw two fair cubical dice simultaneously.

The score for each child is the sum of the two numbers shown on their respective dice.

(a) Complete the table below for the sum of two fair cubical dice below. **1**

	1	2	3	4	5	6
1						
2						
3						
4						
5						
6						

(b) Calculate the probability that Jack obtains a score of 9. **1**

(c) Calculate the probability that Jack and Jill each obtain a score of 9. **1**

(d) By considering the symmetry of the table in (a), show that the probability of Jack and Jill obtaining the same score is $\frac{73}{648}$. **2**

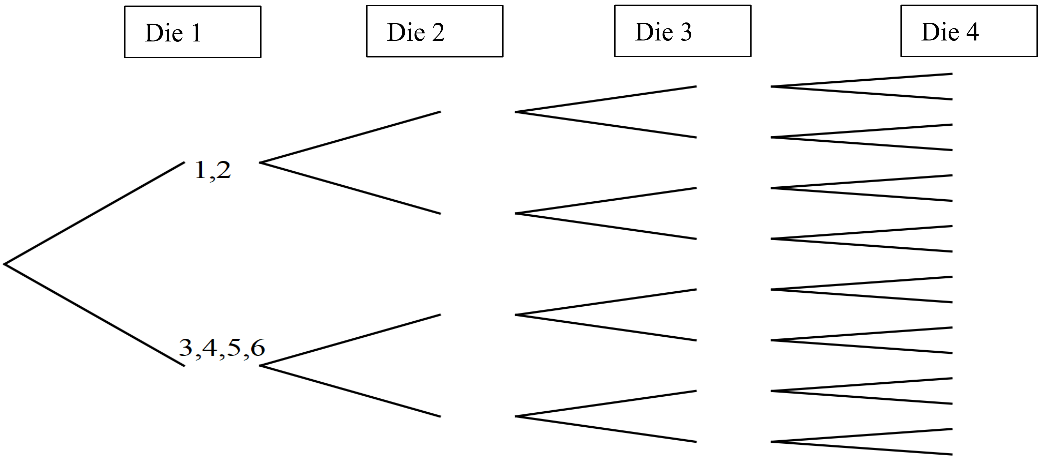
(e) Find the probability that Jack's score exceeds Jill's score. **2**

Question 26 continues on page 27

Question 26 (13 marks) continued

The discrete random variable X denotes the largest number shown on the four dice thrown.

(f) Using the tree diagram, or otherwise show that $P(X \leq 2) = \frac{1}{81}$. **1**



(g) It is known that $P(X \leq x) = \left(\frac{x}{6}\right)^4$, for $x = 1, 2, ..., 6$. **2**

Complete the following probability distribution table.

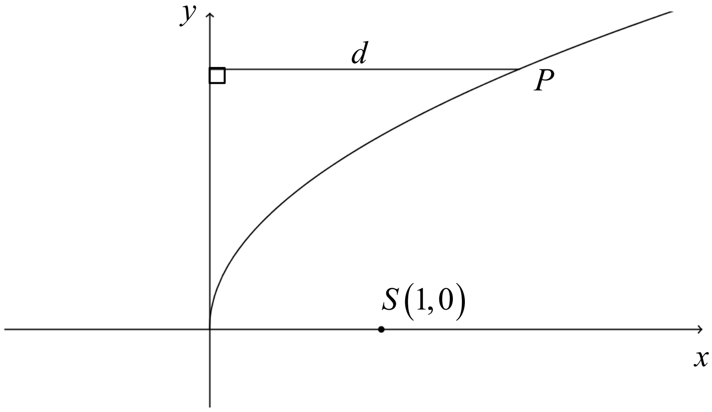
x	1	2	3	4	5	6
$P(X = x)$	$\frac{1}{1296}$	$\frac{15}{1296}$		$\frac{175}{1296}$	$\frac{369}{1296}$	$\frac{671}{1296}$
x^2			9			
$x^2 P(X = x)$			$\frac{585}{1296}$		$\frac{9225}{1296}$	

(h) Calculate $E(X)$, correct to 2 decimal places. **1**

(i) Find $Var(X)$, correct to 2 decimal places. **2**

Question 27 **(7 marks)**

The diagram shows a part of the curve with equation $x = y^2$ and a fixed point $S(1,0)$.
Point P lies on the curve and has y-coordinate k, where $k \geq 0$.



NOT TO SCALE

The distance of P from the y-axis is d, and r is the ratio $\dfrac{d}{SP}$

(a) Show that $r = \dfrac{k^2}{\sqrt{k^4 - k^2 + 1}}$ **2**

Question 27 continues on page 29

(b) Show that $\frac{dr}{dk} = \frac{2k - k^3}{\left(k^4 - k^2 + 1\right)^{\frac{3}{2}}}$. **2**

(c) By solving $\frac{dr}{dk} = 0$, find and verify the value of k that gives the maximum of r. **2**

(d) Hence, find the exact maximum value of r. **1**

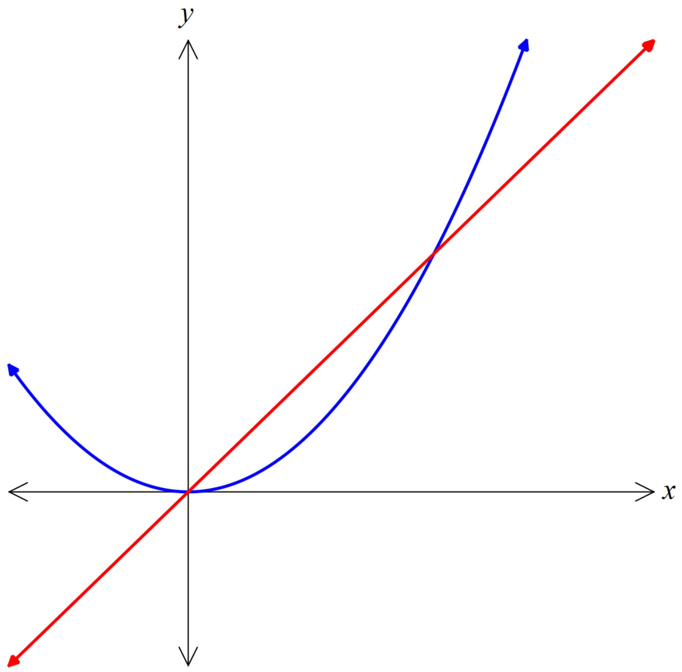
Question 28 (4 marks)

Find the coordinates of any horizontal points of inflexion on the curve with equation **4**

$y = x^4 - 8x^3 + 18x^2 + 4$

Question 29 (3 marks)

The area enclosed between the curve $y = x^2$ and the line $y = mx$ is $\frac{32}{3}$ square units. **3**

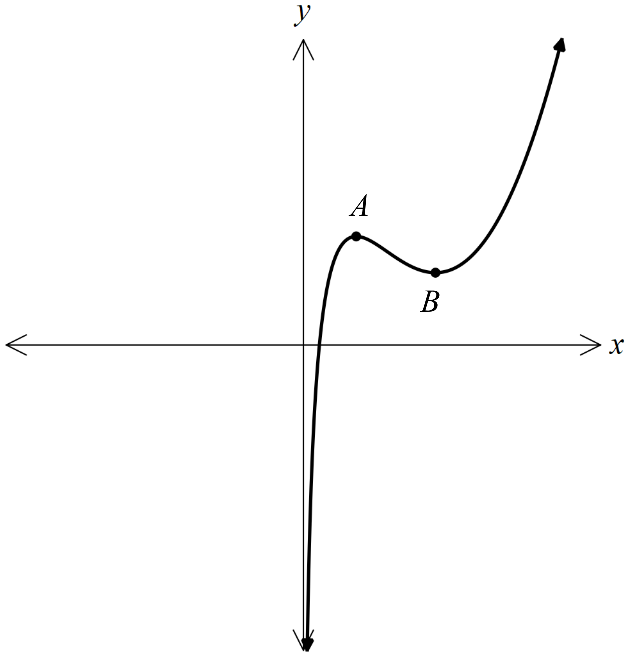


NOT TO
SCALE

Find the value of m if $m > 0$.

Question 30 (4 marks)

The diagram shows part of the curve with the equation $y = f(x)$, where $f(x) = x^2 - 7x + 5\log_e x + 8$, $x > 0$.



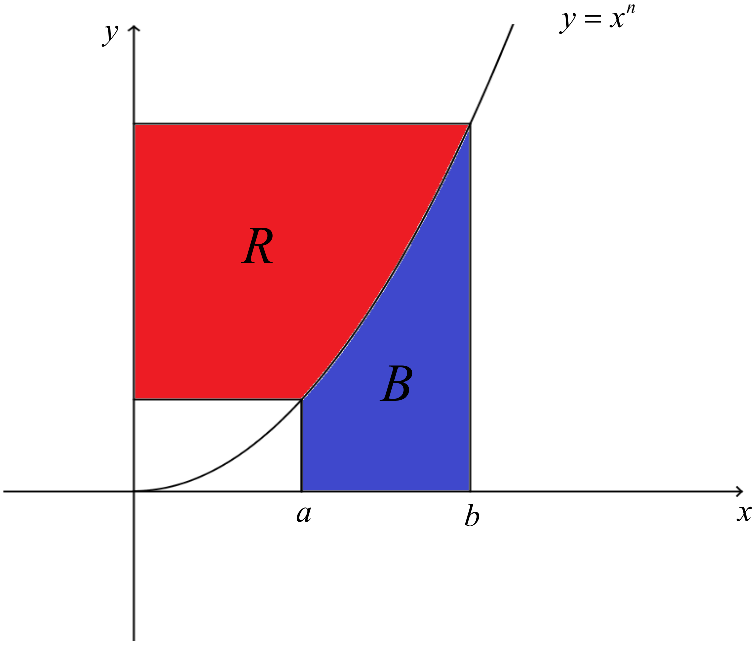
The points *A* and *B* are stationary points of the curve.

(a) Using calculus, find the coordinates of the points *A* and *B* in exact form. **2**

(b) State the coordinates of A_1 which is the stationary point of $y = -2f(x-1)$ which relates to stationary point *A* in $y = f(x)$ and state the nature of A_1. **2**

Question 31 (4 marks)

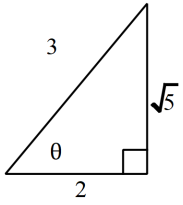
The diagram shows the graph of $y = x^n$, $x \geq 0$ for $n > 1$. **4**



If the red area is three times larger than the blue area, find the value of *n*.

END OF EXAMINATION

Year 12 Mathematics Advanced
Trial Examination 2021
Solutions/Marking Criteria

Question	Solution	Mark	Marking Criteria
1	(B)	1	
2	(A) θ is in the third quadrant, so tan is positive $\tan\theta = \frac{\sqrt{5}}{2}$  	1	
3	(D) Algebraically: $\lvert x+2 \rvert > 4$ $x+2>4 \; or \; x+2<-4$ $x>2 \; or \; x<-6$	1	
4	(B) $T_3 = 8$ $T_6 = 23$ $8 = a+2d$ $23 = a+5d$ $3d = 15$ $d = 5$ $a = -2$	1	

Question	Solution	Mark	Marking Criteria
5	(C) $g(2) = 2^3$ $= 8$ $f(8) = 3\times 8+1$ $= 25$	1	
6	From graph, want values of *x* where graph has positive *y*-values. (C)	1	
7	(A)	1	
8	(C) $y = x\left(x^2+1\right)^3$ $\frac{dy}{dx} = 1\left(x^2+1\right)^3 + x\times 3\left(x^2+1\right)^2 \times 2x$ $= \left(x^2+1\right)^3 + 6x^2\left(x^2+1\right)^2$	1	
9	(A) $\int_{-2}^{6} f(x) = \frac{1}{2}\pi\times 4^2 - \frac{1}{2}(4+2)$ $= 2\pi - 3$	1	
10	(C)	1	

Question	Solution	Mark	Marking Criteria
11	$\frac{\sqrt{5}+1}{\sqrt{5}-2}=\frac{\sqrt{5}+1}{\sqrt{5}-2}\times\frac{\sqrt{5}+2}{\sqrt{5}+2}$ $=\frac{5+2\sqrt{5}+\sqrt{5}+2}{5-2^2}$ $=7+3\sqrt{5}$	2	1 mark for multiplying by conjugate and attempting to expand 1 mark for expanding and simplifying correctly Generally, well done. Some students mixed up their conjugate and used a – sign rather than the + sign
12	$f(x)=e^{2x}$ $f'(x)=2e^{2x}$ $f\left(\frac{1}{2}\right)=e$ $f'\left(\frac{1}{2}\right)=2e$ Equation of tangent: $y-e=2e\left(x-\frac{1}{2}\right)$ $y-e=2ex-e$ $y=2ex$	2	1 mark for finding gradient of tangent 1 mark for finding equation for tangent Generally, well done. Some students used the general result f'(x) for the gradient instead of using f'(1/2)=2e
13(a)	$f(x)=\tan 2x$ $f'(x)=2\sec^2 2x$	1	1 mark for correct derivative Generally, well done.
13(b)	$f(x)=\frac{e^x}{\cos x}$ $f'(x)=\frac{\cos x\times e^x-e^x\times -\sin x}{\cos^2 x}$ $=\frac{e^x(\cos x+\sin x)}{\cos^2 x}$	2	1 mark for correct first part of quotient rule vu' and v^2 1 mark for correct second part of quotient rule uv' Generally, well done. Some students mixed up their quotient rule. Some only did part of it.

Question	Solution	Mark	Marking Criteria
14(a)	$\int \frac{x^4+3x^2+1}{x^2}dx$ $=\int \left(x^2+3+x^{-2}\right)dx$ $=\frac{x^3}{3}+3x-\frac{1}{x}+C$	2	1 mark for simplifying integrated before integrating. 1 mark for correct integral from simplified expression. Also generally well done. Some students did not simplify the fraction and made a mess of the question. Some differentiated some or all of the terms.
14(b)	$\int \frac{4x}{x^2+1}dx=\frac{1}{2}\int \frac{2x}{x^2+1}dx$ $=\frac{1}{2}\log_e\left\|x^2+1\right\|+C$ $=\frac{1}{2}\log_e\left(x^2+1\right)+C$ Should be 2x not ½ x	2	1 mark for recognising factor of 2 1 mark for correct integral. Well done question Some students mixed up their factor and wrote ½ in front rather than 2 in front. Some extension students thought it was an inverse tan result.
15	$S_{12}=360$ $T_{12}=2T_1$ $S_n=\frac{n}{2}(a+l)$ $360=\frac{12}{2}(a+2a)$ $60=3a$ $a=20$ Therefore smallest angle is $20°$.	3	1 mark for recognising $S_{12}=360$ and using 1 mark for relating T_1 and T_{12} and using relationship 1 mark for finding correct smallest angle Generally well done by students who recognised T12.was 2a

Question	Solution	Mark	Marking Criteria
16(a)	$\log_2 x+\log_2(x-3)=2$ $\log_2(x(x-3))=2$ $4=x(x-3)$ $0=x^2-3x-4$	2	1 mark for applying one or more log laws correctly 1 mark for changing equation from logarithmic equation to quadratic, showing adequate working. Generally well done for the most. Some students need to learn their log laws better.
16(b)	$0=(x-4)(x-1)$ $x=4\ or\ x=1$ $but\ x>0$ $x=4$	1	1 mark for solving quadratic and identifying only one solution fits in domain of equation. A lot got this wrong because they didn't consider the natural domain of the log functions and included an the incorrect answer x=1.
17(a)	$\angle COQ=\pi-\frac{\pi}{2}-\frac{\pi}{6}$ $=\frac{\pi}{3}$ $l_{CQ}=2\times\frac{\pi}{3}$ $=\frac{2\pi}{3}cm$	2	1 mark for $\angle COQ$ 1 mark for arc length. Some students used $l=\frac{60}{360}\times 2\pi r$ $l=\frac{1}{6}\times 2\pi\times 2$ $l=\frac{4\pi}{6}$ $l=\frac{2\pi}{3}$ Which is fine and demonstrates understanding but it is less efficient
17(b)	$A=\frac{1}{2}r^2\theta$ $=\frac{1}{2}\times 7^2\times\frac{\pi}{6}$ $=\frac{49\pi}{12}\text{ cm}^2$	2	1 mark for substituting into formula correctly (recognising radius) 1 mark for simplifying to give area in exact form. Several students did not know this formula and obviously did not realise that it is on the reference sheet.

Question	**Solution**	**Mark**	**Marking Criteria**
17(c)	$AP^2 = OP^2 + OA^2 - 2 \times OP \times OA \cos \angle AOP$ $= 7^2 + 7^2 - 2 \times 7 \times 7 \times \cos \frac{\pi}{6}$ $= 49 + 49 - 98 \times \frac{\sqrt{3}}{2}$ $AP = \sqrt{98 - 49\sqrt{3}}$ $= 3.6234\ldots \text{cm}$ =3.6 cm (1*dp*)	2	1 mark for substituting into cosine rule correctly. 1 mark for evaluating cosine expression to get answer. Some students found the length of the circular arc $AP = r\theta = 7 \times \frac{\pi}{6} \approx 3.66519129$ instead of the interval *AP* which was not awarded any marks. Q17 was reasonably well done
18(a)	$P(B) = 0.3$ $P(B \cap T) = x$	1	1 mark for completing each statement **Q18 was poorly understood**
18(b)	$1 = 0.5 + P(B) + P(T) - P(B \cap T)$ $0.5 = 0.3 + P(T) - x$ $P(T) = 0.2 + x$ $\therefore P(T \cap \overline{B}) = 0.2$	1	1 mark for finding probability
18(c)	$P(B \mid T) = \frac{P(B \cap T)}{P(T)}$ $0.5 = \frac{x}{0.2 + x}$ $0.1 + \frac{x}{2} = x$ $\frac{x}{2} = 0.1$ $x = 0.2$ $P(B \cap T) = 0.2$	2	1 mark for recognising and applying formula for conditional probability, substituting in probabilities. 1 mark for finding probability. Many students did not recognise this as conditional probability. Some tried to use the formula but misquoted the denominator as *P(B)*. Others oversimplified *P(T)* as just 0.2 instead of 0.2+*x*.

Question	Solution	Mark	Marking Criteria
18(d)	$P(B \cap \overline{T}) = 0.3 - x$ $= 0.3 - 0.2$ $= 0.1$	1	1 mark for probability Most students used their answer from part c correctly to find an answer for part d
19(a)	Using calculator, $r = -0.854...$ $= -0.85\ (2dp)$	1	1 mark for correlation coefficient. Reasonably well done. A few students need to learn how to use their calculators.
19(b)	Using calculator: $y = A + BX$ $y = -57.98x + 863.72\ (2dp)$	2	1 mark for A and B 1 mark for A and B in correct position in linear equation
19(c)	Let $x = 5.5$ $y = -57.98 \times 5.5 + 863.72$ $= 544.83$ Estimated income per capita is \$545 (nearest dollar)	1	1 mark for finding y given x and using equation.
20(a)	Let $\dot{x} = 0$ $0 = 4 - 8\cos t$ $8\cos t = 4$ $\cos t = \frac{1}{2}$ $t = \frac{\pi}{3}, 2\pi - \frac{\pi}{3}$ $= \frac{\pi}{3}, \frac{5\pi}{3}$ The first two times the particle is at rest is $\frac{\pi}{3}$ seconds and $\frac{5\pi}{3}$ seconds.	2	1 mark for getting to $\cos t = \frac{1}{2}$ and recognising first quadrant solution 1 mark for other solution. Many students answered in degrees rather than radians. Time, $t = 60^\circ, 300^\circ$ seconds has no meaning whereas $t = \frac{\pi}{3}, \frac{5\pi}{3} = 1.047, 5.236$ seconds does have meaning. Answers MUST be in radians.

Question	Solution	Mark	Marking Criteria
20(b)	$x=\int(4-8\cos t)dt$ $=4t-8\sin t+C$ When $t=0,\ x=0$ $\therefore C=0$ $x=4t-8\sin t$	1	1 mark for correct displacement equation Many students did not include +c or show working for c=0. The mark was awarded this time but students should show full working every time.
20(c)	When $t=\frac{\pi}{3}$ $x=4\times\frac{\pi}{3}-8\sin\frac{\pi}{3}$ $=\frac{4\pi}{3}-8\times\frac{\sqrt{3}}{2}$ $=\frac{4\pi}{3}-4\sqrt{3}$ When $t=\frac{5\pi}{3}$ $x=4\times\frac{5\pi}{3}-8\sin\frac{5\pi}{3}$ $=\frac{20\pi}{3}+8\times\frac{\sqrt{3}}{2}$ $=\frac{20\pi}{3}+4\sqrt{3}$ $\text{distance}=\frac{20\pi}{3}+4\sqrt{3}-\left(\frac{4\pi}{3}-4\sqrt{3}\right)$ $=\frac{16\pi}{3}+8\sqrt{3}\ metres$	2	1 mark for one correct displacement for one of the times. 1 mark for final distance travelled in exact or non-exact form. Students must work in radians for this question. It does not make sense to write statements like $4\times 60^\circ$

Question	Solution	Mark	Marking Criteria
21(a)	$P = 20 + 10e^{\frac{t}{50}}$ $\frac{dP}{dt} = 10 \times \frac{1}{50} e^{\frac{t}{50}}$ $= \frac{1}{5} e^{\frac{t}{50}}$ When $t = 0$, $\frac{dP}{dt} = \frac{1}{5} e^{0}$ $= \frac{1}{5}$ Initially, population is increasing at a rate of 200 people per year.	2	1 mark for differentiating correctly. 1 mark for rate interpreted correctly (people per year). A number of students followed all steps correctly but did not actually interpret their result a rate. (increasing at a rate of 200 people per year). Some students, maybe through pattern of usual questions, found the initial population rather than the original rate. A few students struggled to differentiate $\frac{d}{dt}\left(e^{\frac{t}{50}}\right) = \frac{1}{50} e^{\frac{t}{50}}$
21(b)	Let $t = 20$ $P = 20 + 10e^{\frac{20}{50}}$ $= 34.9182...$ The predicted population on the 1st January 2040 will be 35 000.	1	1 for correct predicted population This question was done very well. Please make sure you take note of rounding.
21(c)	Let $P = 40$ $40 = 20 + 10e^{\frac{t}{50}}$ $e^{\frac{t}{50}} = 2$ $\frac{t}{50} = \ln 2$ $t = 50 \ln 2$ $= 34.657...$ Predicted population will reach 40 000 during 2054	2	1 mark for substituting and starting process of solving equation 1 mark for correct value year?. Some students did not recognise to substitute 40 rather than 40 000. Make sure that you show your calculator output 34.657… before rounding. If you did not do this and indicated 35 years that would be mean during/on beginning of 2055 which was incorrect. Mark was awarded for students who left decimal but did not interpret correctly.

Question	Solution	Mark	Marking Criteria
22(a)	$amplitude = \frac{6-2}{2}$ $= 2$ $\therefore A = 2$ $\frac{2\pi}{b} = 8$ $b = \frac{\pi}{4}$	2	1 mark for at least one value right If all values correct, give two marks. Finding A was done well. Finding b was poorly done. Students need to know that $period = \frac{2\pi}{b}$

Question	Solution	Mark	Marking Criteria
22(b)	$h=4-2\sin\frac{\pi}{4}t$ $5=4-2\sin\frac{\pi t}{4}$ $\sin\frac{\pi}{4}t=\frac{-1}{2}$ $\frac{\pi}{4}t=\pi+\frac{\pi}{6},2\pi-\frac{\pi}{6}$ $\frac{\pi}{4}t=\frac{7\pi}{6},\frac{11\pi}{6}$ $t=\frac{28}{6},\frac{44}{6}$ $=4\text{h }40\text{ min}$ First Time at 4:40 am.	2	1 mark for setting up equation 1 mark for correct answers to equation. Most students sent up an inequation rather than equation – which would give the correct solution but means some incorrect statements were made. Some students struggled with dealing with trig equation using compound angle $\frac{\pi}{4}t$. Some students mixed up radians and degrees . Students need to make sure they read and answer the question – it wanted the specific time.
23	$\int_0^2\sqrt{x}dx\approx\frac{0.5}{2}\left(\sqrt{0}+\sqrt{2}+2\left(\sqrt{\frac{1}{2}}+\sqrt{1}+\sqrt{\frac{3}{2}}\right)\right)$ $=1.8194...$ $=1.82\ (2dp)$	2	1 for setting up table or substituting correctly into trapezoidal rule 1 mark for correct answer. Generally well done if student knew formula. Main error was dealing with four sub intervals – it is suggested that you draw a number line and divide up to check that you are correct. Also, preferreable to leave exact in the formula where manageable and then round at end.
24(a)	$2\cos^2 x=4-5\sin x$ $2\left(1-\sin^2 x\right)=4-5\sin x$ $2-2\sin^2 x=4-5\sin x$ $2\sin^2 x-5\sin x+4-2=0$ $2\sin^2 x-5\sin x+2=0$	1	1 for replacing $\cos^2$ and getting to required equation Very well done – Some students did LHS and RHS which was not necessary and led to a less elegant solution.

Question	**Solution**	**Mark**	**Marking Criteria**
24(b)	$2\cos^2 x = 4-5\sin x$ $2\sin^2 x - 5\sin x + 2 = 0$ $2\sin^2 x - 4\sin x - \sin x + 2 = 0$ $2\sin x(\sin x - 2) - 1(\sin x - 2) = 0$ $(2\sin x - 1)(\sin x - 2) = 0$ $\sin x = \frac{1}{2} \quad or \quad \sin x = 2 \ but \ -1 \le \sin x \le 1$ $\sin x = \frac{1}{2}$ $x = 30°, 150°$	2	1 for finding quadratic equation in terms of sin 1 for finding two solutions. Generally well done. Students should give reason why sinx=2 does not give solution, or some statement to that affect. The domain is given in degrees so all working should be in degrees, including final answer.
25(a)	When $x = 2$ $y = 24(2-1)$ $y = 24$ The point of intersection is $(2, 24)$ Sub into $y = ax^3 + bx^2 + 4$ $24 = a \times 2^3 + b \times 2^2 + 4$ $24 = 8a + 4b + 4$ $20 = 8a + 4b$ $5 = 2a + b \ldots(1)$	2	1 mark for finding *y*-value 1 mark for correctly showing derivation of required equation. Generally well done, if question interpeted correctly.
25(b)	Gradient of tangent is 24 $\frac{dy}{dx} = 3ax^2 + 2bx$ $24 = 3a \times 2^2 + 2b \times 2$ $24 = 12a + 4b$ $6 = 3a + b \ldots (2)$ (2)-(1):	2	1 mark is for differenating and setting equal to 24 1 mark for finding a and b Generally well done.

<table>
<tr><th>Question</th><th>Solution</th><th>Mark</th><th>Marking Criteria</th></tr>
<tr><td></td><td>$a=1$
$\therefore b=3$</td><td></td><td></td></tr>
<tr><td>26(a)</td><td>
<table>
<tr><th></th><th>1</th><th>2</th><th>3</th><th>4</th><th>5</th><th>6</th></tr>
<tr><td>1</td><td>2</td><td>3</td><td>4</td><td>5</td><td>6</td><td>7</td></tr>
<tr><td>2</td><td>3</td><td>4</td><td>5</td><td>6</td><td>7</td><td>8</td></tr>
<tr><td>3</td><td>4</td><td>5</td><td>6</td><td>7</td><td>8</td><td>9</td></tr>
<tr><td>4</td><td>5</td><td>6</td><td>7</td><td>8</td><td>9</td><td>10</td></tr>
<tr><td>5</td><td>6</td><td>7</td><td>8</td><td>9</td><td>10</td><td>11</td></tr>
<tr><td>6</td><td>7</td><td>8</td><td>9</td><td>10</td><td>11</td><td>12</td></tr>
</table>
</td><td>1</td><td>1 for completing table
Completed well</td></tr>
<tr><td>26(b)</td><td>$P(\text{Jack gets } 9)=\frac{4}{36}$
$=\frac{1}{9}$</td><td>1</td><td>1 mark for correct probability
Completed well</td></tr>
<tr><td>26(c)</td><td>$P(\text{Jack\&Jill score } 9)=P(\text{Jack scores } 9)\times P(\text{Jill scores } 9)$
$=\frac{1}{9}\times\frac{1}{9}$
$=\frac{1}{81}$</td><td>1</td><td>1 mark for correct probability
Completed well although some students added the probabilities instead of multiplying them.</td></tr>
</table>

Question	Solution	Mark	Marking Criteria
26(d)	$P(\text{Jack\&Jill same score})$ $=P(2,2)+P(3,3)+P(4,4)+P(5,5)+P(6,6)+$ $P(7,7)+P(8,8)+P(9,9)+P(10,10)+P(11,11)+P(12,12)$ $=2P(2,2)+2P(3,3)+2P(4,4)+2P(5,5)+2P(6,6)+P(7,7)$ $=P(7,7)+2\left[P(2,2)+P(3,3)+P(4,4)+P(5,5)+P(6,6)\right]$ $=\left(\frac{6}{36}\right)^2+2\left[\left(\frac{1}{36}\right)^2+\left(\frac{2}{36}\right)^2+\left(\frac{3}{36}\right)^2+\left(\frac{4}{36}\right)^2+\left(\frac{5}{36}\right)^2\right]$ $=\frac{73}{648}$	2	1 mark for some attempt to work towards finding probability Full marks for adequate demonstrating/showing of probability. Not completed well. In fact, many students didn't attempt it. Also, a few students didn't use symmetry which the question asked for and used values without any explanation.
26(e)	$1=P(\text{Jack>Jill})+P(\text{Jack<Jill})+P(\text{Jack=Jill})$ $1=2P(\text{Jack>Jill})+\frac{73}{648}$ $2P(\text{Jack>Jill})=1-\frac{73}{648}$ $P(Jack>Jill)=\frac{575}{1296}$	2	1 mark for setting up equation, or using part before to attempt to find probability 1 mark for final correct answer. Not completed well. In fact, many students didn't attempt it.
26(f)	From diagram, $P(1\ or\ 2\text{ on first die})=\frac{2}{6}$ $P(1\text{ or 2 on every die})=\frac{2}{6}\times\frac{2}{6}\times\frac{2}{6}\times\frac{2}{6}$ $=\frac{1^{4}}{3}$ $=\frac{1}{81}$	1	1 mark for adequate justification. Completed well.

<table>
<tr><th>Question</th><th>Solution</th><th>Mark</th><th>Marking Criteria</th></tr>
<tr><td>26(g)</td><td><table>
<tr><td>x</td><td>1</td><td>2</td><td>3</td><td>4</td><td>5</td><td>6</td></tr>
<tr><td>$P(X=x)$</td><td>$\frac{1}{1296}$</td><td>$\frac{15}{1296}$</td><td>$\frac{65}{1296}$</td><td>$\frac{175}{1296}$</td><td>$\frac{369}{1296}$</td><td>$\frac{671}{1296}$</td></tr>
<tr><td>x^2</td><td>1</td><td>4</td><td>9</td><td>16</td><td>25</td><td>36</td></tr>
<tr><td>$x^2P(X=x)$</td><td>$\frac{1}{1296}$</td><td>$\frac{60}{1296}$</td><td>$\frac{585}{1296}$</td><td>$\frac{2800}{1296}$</td><td>$\frac{9225}{1296}$</td><td>$\frac{24156}{1296}$</td></tr>
</table></td><td>2</td><td>1 mark for some correct values
2 marks for all correct values
Completed well.</td></tr>
<tr><td>26(h)</td><td>$$E(X)=1\times\frac{1}{1296}+2\times\frac{15}{1296}+3\times\frac{65}{1296}+4\times\frac{175}{1296}+5\times\frac{369}{1296}+6\times\frac{671}{1296}=5.24\ (2dp)$$</td><td>1</td><td>1 mark for correct expected value.
Not completed well. I strongly advise students double check their answer when complex fractions are involved.</td></tr>
<tr><td>26(i)</td><td>$$Var(X)=E(X^2)-5.24^2 =\frac{1}{1296}+\frac{60}{1296}+\frac{585}{1296}+\frac{2800}{1296}+\frac{9225}{1296}+\frac{24156}{1296}-5.24^2 =0.96\ (2dp$$</td><td>2</td><td>1 mark for some correct working to calculate variance
1 mark for correct value of variance
Not completed well. Many students forgot to square the expected value.</td></tr>
</table>

Question	Solution	Mark	Marking Criteria
27(a)	$P\left(k^2,k\right)$ $\therefore d=k^2$ $PS^2=\left(x_2-x_1\right)^2+\left(y_2-y_1\right)^2$ $=\left(k^2-1\right)^2+\left(k-0\right)^2$ $=k^4-2k^2+1+k^2$ $=k^4-k^2+1$ $r=\dfrac{d}{PS}$ $=\dfrac{k^2}{\sqrt{k^4-k^2+1}}$	2	1 mark for recognising expression for d 1 mark for correctly showing expression for PS . Completed well.
27(b)	$v=\left(k^4-k^2+1\right)^{\frac{1}{2}}$ $u=k^2$ $\dfrac{du}{dk}=2k$ $v'=\dfrac{1}{2}\left(k^4-k^2+1\right)^{\frac{-1}{2}}\times\left(4k^3-2k\right)$ $=\dfrac{4k^3-2k}{2\sqrt{k^4-k^2+1}}$	2	1 for applying quotient rule correctly 1 mark for simplifying expression to get required result Not completed well. The biggest issue with this question was that there was insufficient space to answer it resulting in students squashing their solution in the allocated space. The students who used the extra writing space were able to set out their solution more clearly and consequently more of them got it correct.

Question	Solution	Mark	Marking Criteria
	$\frac{dr}{dk}=\frac{\sqrt{k^4-k^2+1}\times 2k-k^2\times\frac{4k^3-2k}{2\sqrt{k^4-k^2+1}}}{\left(\sqrt{k^4-k^2+1}\right)^2}$ $=\frac{4k\left(\sqrt{k^4-k^2+1}\right)^2-k^2\left(4k^3-2k\right)}{\left(k^4-k^2+1\right)\times 2\sqrt{k^4-k^2+1}}$ $=\frac{4k\left(k^4-k^2+1\right)-k^2\left(4k^3-2k\right)}{2\left(k^4-k^2+1\right)^{\frac{3}{2}}}$ $=\frac{4k^5-4k^3+4k-4k^5+2k^3}{2\left(k^4-k^2+1\right)^{\frac{3}{2}}}$ $=\frac{4k-2k^3}{2\left(k^4-k^2+1\right)^{\frac{3}{2}}}$ $=\frac{2k-k^3}{\left(k^4-k^2+1\right)^{\frac{3}{2}}}$		
27(c)	Let $\frac{dr}{dk}=0$ $0=\frac{4k\left(k^4-k^2+1\right)-k^2\left(4k^3-2k\right)}{\left(k^4-k^2+1\right)^{\frac{3}{2}}}$ $0=4k^5-4k^3+4k-4k^5+2k^3$ $0=4k-2k^3$ $0=2k\left(2-k^2\right)$ $\therefore k=\sqrt{2} \quad k>0$	2	1 mark for solving equation 1 mark for verifying that the value gives a maximum. Completed well. However quite a few students wrote: $k^2 = 2$ $k = 2$!!!! Also, some students forgot to test for max/min. In addition, some students forgot that k > 0.

<table>
<tr><th>Question</th><th>Solution</th><th>Mark</th><th>Marking Criteria</th></tr>
<tr><td></td><td>
<table>
<tr><td>k</td><td>1</td><td>$\sqrt{2}$</td><td>2</td></tr>
<tr><td>$\frac{dr}{dk}$</td><td>1</td><td>0</td><td>$-0.85\ldots$</td></tr>
</table>
Therefore $k=\sqrt{2}$ gives maximum.</td><td></td><td></td></tr>
<tr><td>27(d)</td><td>$k=\sqrt{2}$
When $r=\frac{\left(\sqrt{2}\right)^2}{\sqrt{\left(\sqrt{2}\right)^4-\left(\sqrt{2}\right)^2+1}}$
$=\frac{2}{\sqrt{4-2+1}}$
$=\frac{2}{\sqrt{3}}$</td><td>1</td><td>1 mark for finding r from result in (c)
Completed well</td></tr>
<tr><td>28</td><td>$y=x^4-8x^3+18x^2+4$
$\frac{dy}{dx}=4x^3-24x^2+36x$
$\frac{d^2y}{dx^2}=12x^2-48x+36$
For possible points of inflection, let $\frac{d^2y}{dx^2}=0$
$0=12x^2-48x+36$
$x^2-4x+3=0$
$(x-3)(x-1)=0$
$\therefore x=1 \text{ or } x=3$
For horizontal point of inflection, value of x will also be stationary point
When $x=1$,</td><td>4</td><td>1 mark for differentiating twice
1 mark for finding values of x that make second derivative zero.
1 mark for finding which value is also a stationary point and y-value.
1 value for checking that it changes concavity.
Many students didn’t test for change of concavity with table of values.
Some students didn’t exclude one of the POIs which is not horizontal.</td></tr>
</table>

<table>
<tr><th>Question</th><th>Solution</th><th>Mark</th><th>Marking Criteria</th></tr>
<tr><td></td><td>

$\frac{dy}{dx}=4(1)^3-24(1)^2+36(1)$

$=16$

When $x=3$,

$\frac{dy}{dx}=4(3)^3-24(3)^2+36(3)$

$=0$

Check for change of concavity:

<table>
<tr><td>x</td><td>2.9</td><td>3</td><td>3.1</td></tr>
<tr><td>$\frac{d^2y}{dx^2}$</td><td>$\frac{-57}{25}$</td><td>0</td><td>$\frac{63}{25}$</td></tr>
</table>

When $x=3$,

$y=3^4-8(3)^3+18(3)^2+4$

$=31$

$\therefore (3,31)$ is a horizontal point of inflection.

</td><td></td><td></td></tr>
<tr><td>29</td><td>

Let the x-value of the point of intersection by a

Point of intersection: (a,a^2) from $y=x^2$ and (a,ma) from $y=mx$

$\therefore ma=a^2$

$m=a$

</td><td>3</td><td>

1 mark for setting up area integral

1 mark for finding primitive of difference

1 mark for finding the correct value of a.

General well done.
Some students didn’t find m=a.
Some students were not able to find the correct primitive.

</td></tr>
</table>

Question	Solution	Mark	Marking Criteria
	$\frac{32}{3}=\int_0^a \left(mx-x^2\right)dx$ $\frac{32}{3}=\left[\frac{mx^2}{2}-\frac{x^3}{3}\right]_0^a$ $\frac{32}{3}=\frac{ma^2}{2}-\frac{a^3}{3}$ $64=3ma^2-2a^3$ $64=3a^3-2a^3$ since $m=a$ $a^3=64$ $a=4$		
30(a)	$f\left(x\right)=x^2-7x+5\log_e x+8$ $f'\left(x\right)=2x-7+\frac{5}{x}$ Let $f'\left(x\right)=0$ $0=2x-7+\frac{5}{x}$ $0=2x^2-7x+5$ $0=2x^2-2x-5x+5$ $0=2x\left(x-1\right)-5\left(x-1\right)$ $0=\left(2x-5\right)\left(x-1\right)$ $x=1\ or\ x=\frac{5}{2}$ $f\left(1\right)=\left(1\right)^2-7\left(1\right)+5\log_e 1+8$ $=2$ $\therefore A\left(1,2\right)$	2	1 mark finding stationary points. 1 mark for correct coordinates for A and B. Generally well done.

Question	Solution	Mark	Marking Criteria
	$f\left(\frac{5}{2}\right)=\left(\frac{5}{2}\right)^2-7\left(\frac{5}{2}\right)+5\log_e\left(\frac{5}{2}\right)+8$ $=5\log_e\left(\frac{5}{2}\right)-\frac{13}{4}$ $\therefore B\left(\frac{5}{2},5\log_e\left(\frac{5}{2}\right)-\frac{13}{4}\right)$		
30(b)	The graph of $y=-2f(x-1)$ is the graph of $f(x)$ shifted 1 unit right then vertically dilated by a factor of -2 . A_1 becomes a local minimum. $A_1=(2,-4)$	2	1 mark for coordinates of A 1 mark for correct nature. Some students were not able to find the point by transformation. Some students didn't specify the nature of the point.
31	Let B be the area of the blue region Let W be the area of the small rectangle Let R b e the area of the big rectangle $B=\int_a^b x^n dx$ $=\frac{1}{n+1}\left[x^{n+1}\right]_a^b$ $=\frac{1}{n+1}\left(b^{n+1}-a^{n+1}\right)$ $W=a\times a^n$ $=a^{n+1}$ $R=b\times b^n$	4	1 mark for blue area 1 mark for rectangle areas 1 mark for setting up equation with relationship between areas 1 mark for correct value. Some students not able to build the relationship between the areas using rectangles. Some students not able to solve the equation.

Question	Solution	Mark	Marking Criteria
	$R = B + 3B + W$ $= 4B + W$ $b^{n+1} = \frac{4}{n+1}\left(b^{n+1} - a^{n+1}\right) + a^{n+1}$ $b^{n+1} - a^{n+1} = \frac{4}{n+1}\left(b^{n+1} - a^{n+1}\right)$ $1 = \frac{4}{n+1}$ $n + 1 = 4$ $n = 3$		